Multi-CRC Polar Codes and Their Applications

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***Abstract*—Polar codes under successive cancelation list (SCL) decoding are capable of achieving almost the same or better performance than turbo codes or low density parity-check codes with the help of single cyclic redundancy check (CRC). This decoding scheme, however, suffers from very high complexity with long delay and large memory space. Motivated by this research problem, we propose a novel coding scheme called multi-CRC polar code for significant reduction of memory size and decoding delay but with negligible performance loss. Our analysis and simulation have shown that about half reduction of memory size and decoding delay can be achieved in SCL decoding. We also apply this scheme to hybrid automatic repeat request (HARQ) system to aid retransmission and show that the throughput of multi-CRC polar code is higher than that of the single-CRC one.**

***Index Terms*—Polar codes, CRC, SCL decoding, HARQ.**

I. Introduction

POLAR CODES proposed by Arıkan [1] are proven to achieve symmetric capacity of any binary-input discrete memoryless channel (B-DMC) under successive cancellation (SC) decoding. However, the finite length performance of polar codes using SC decoding is not satisfactory. Several alternative decoding schemes using ideas such as belief propagation [2] and linear programming [3], have been proposed to enhance the finite length performance of polar codes. Recently, polar codes under successive cancelation list (SCL) decoding were found to be capable of achieving almost the same performance as (or even better performance than) turbo and low-density parity-check codes with the help of single cyclic redundancy check (CRC) [4], [5]. This letter will address the high decoding complexity as well as long decoding delay (which are pertinent to the existing SCL decoding approach) by proposing a novel low-complexity decoding scheme of polar code but with less than 0.1 dB performance loss.

Specifically, we propose a novel construction of polar codes based on CRC by choosing and outputting decoded bits as early as possible with the aid of multiple CRCs to save memory space and decoding delay. We show that by properly designing, significant reduction of decoding complexity can be achieved with almost the same performance as that of single CRC aided SCL decoding. Moreover, we apply this construction to hybrid automatic repeat request (HARQ) system to aid the retransmission and show that system throughput can be improved.

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The remainder of the letter is organized as follows. Section II gives a review of polar codes and some related works. In Section III, multi-CRC polar codes and the corresponding SCL decoding are proposed, reduction of memory space and decoding delay are analyzed, followed by simulation results for performance evaluation. In Section IV, we apply multi-CRC polar code to HARQ system and verify its throughput improvement. Section V concludes the letter.

II. Polar Codes and SCL Decoding

A. Polar Codes

Polar coding [1] is a code construction method which can achieve the capacity of symmetric B-DMC by using channel combining and splitting. Let $N = 2^n$ and K denote the length of the polar code and that of the information block, respectively. Let $W : X \to Y$ denote an arbitrary B-DMC with input alphabet $X = \{0, 1\}$, output alphabet Y, and transition probabilities $W(y|x)$, $x \in X$, $y \in Y$. Let W^N denote the B-DMC consisting of N independent copies of W. Channel combining means that N copies of a given B-DMC W are combined in a recursive manner to form a vector channel $W_N : X^N \to Y^N$. In contrast, channel splitting refers to splitting W_N into N binary-input channels $\{W_N^{(i)}, i = 1, 2, \ldots, N\}$ which exhibit a polarization effect, meaning that one part of these channels become noise-free while the others turn to be completely noisy when N approaches infinity. Polar encoding consists of sending K information bits over a subset of almost noise-free channels while the remaining $N - K$ channels carry a fixed bit sequence called frozen bits. Let a_1^N denote an arbitrary vector $(a_1, \ldots, a_N)$, and a_i^j denote the subvector $(a_i, \ldots, a_j)$. Polar coding can be expressed as $X_1^N = u_1^N G_N = u_1^N B_N F^{\otimes n}$, where B_N is a bit-reversal permutation matrix, $F = [1\ 0; 1\ 1]$, $F^{\otimes n}$ is the nth Kronecker power of F, and u_1^N denotes the encoding bit sequence. The indices of u_1^N are divided into two subsets $\mathcal{I}$ and $\mathcal{F}$, where $\mathcal{I}$ contains the information bits and $\mathcal{F}$ denotes the complementary set of $\mathcal{I}$. The interested reader may refer to [1] for more details.

B. Decoding of Polar Code

The estimation $\hat{u}_1^N$ of the original input vector u_1^N can be obtained based on the received sequence y_1^N from the channel according to the following method in [1]:

$$\hat{u}_i = \begin{cases} h_i(y_1^N, \hat{u}_1^{i-1}), & if\ i \in \mathcal{I} \\ u_i, & if\ i \in \mathcal{F} \end{cases}$$

where

$$h_i(y_1^N, \hat{u}_1^{i-1}) \triangleq \begin{cases} 0, & \text{if} \frac{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|u_i=0)}{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|u_i=1)} \geq 1 \\ 1, & \text{otherwise.} \end{cases}$$

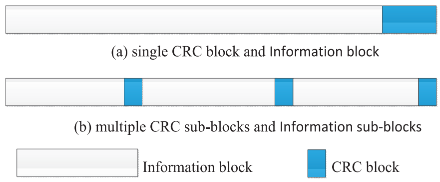

Fig. 1. Structure of input sequence of polar encoder for different schemes.

Polar codes can be decoded with an efficient SC decoder whose idea can be graphically displayed by a path searching procedure over a code tree with depth N. Though the SC decoding method can approach the channel capacity with $O(N \log N)$ complexity, it does not perform well for polar code with small or medium code lengths. In [6], Tal and Vardy proposed SCL decoding in which decoder splits each current path into two, attempting both $u_i = 0$ and $u_i = 1$, and keeps L survival paths where L is also called as list size. Simulations show that polar codes can achieve the maximum likelihood (ML) limit when the signal-noise ratio (SNR) is relatively high. CRC was proposed to further improve the performance of polar codes [5]. It is noted that the CRC aided SCL decoding outperforms the turbo codes and low density parity-check codes. Though the SCL decoder improves the performance of polar codes, its decoding complexity increases with the list size L. To solve this problem, Li developed adaptive successive cancelation list (AD-SCL) decoding in [7]. Instead of setting L to a fixed number, L is adaptive to the channel condition so as to avoid unnecessary calculation and storage while the performance of polar codes can be maintained. In order to reduce the memory space and output delay, we introduce multi-CRC polar code to help SCL decoder select and output the decoded bits as early as possible.

III. Multi-CRC Aided SCL Decoding

In this section, we first propose a novel construction of multi-CRC polar code and its corresponding list decoding. Then we analyze the complexity of SCL decoder with regard to the memory space of survival path and the output delay of decoded bits. Further, we show the performances of multi-CRC based polar code under SCL and AD-SCL decoding by simulating the block error rate (BLER) compared to that of the single CRC aided SCL decoding.

A. Encoding of Multi-CRC Based Polar Codes

Before polar encoding, an information block should be processed by one CRC encoder with r check parity bits for single CRC aided polar code [see Fig. 1(a)]. For a multi-CRC polar code in Fig. 1(b), we split the information block into M sub-blocks, process each sub-block by one CRC encoder with r' check parity bits, where $r' = r/M$, to keep the same code rate. Then we collect them alternatively to form a new sequence which will be sent to the polar encoder.

B. Multi-CRC Aided SCL Decoding

Correspondingly, the SCL decoding scheme should be modified according to Algorithm 1. The decoding procedure is the same as original list decoding except that, when the decoding of any sub-block and its corresponding CRC is accomplished, the

Algorithm 1. Multi-CRC aided SCL decoding

Input:
Position and length of each CRC;
Received vector y_1^N from the channel;
Procedure:
Estimation $\hat{u}_1^N$ of the original input vector u_1^N;
For every input sub-block i of decoder of polar code, do
 for i $\leftarrow$ 1:M;
  for every input information bit j in sub-block i, do
  for j $\leftarrow$ 1:K/M;
   Calculate the likelihood value of the current path based on estimation of previous $i-1$ information sub-blocks;
   if $j = K/M$;
    Apply CRC detection to the current sub-block, select and store the sequence which fulfills the following conditions:
    Condition 1. The corresponding CRC is successful
    Condition 2. This sequence has the largest likelihood value among all sequences fulfilling Condition 1.
    Save the decoded bits as estimation of information sub-block i and as the input of next sub-block decoding;
  end for
end for
return $\hat{u}_1^N$, where $\hat{u}_i = u_i$ if ith element is a frozen bit.

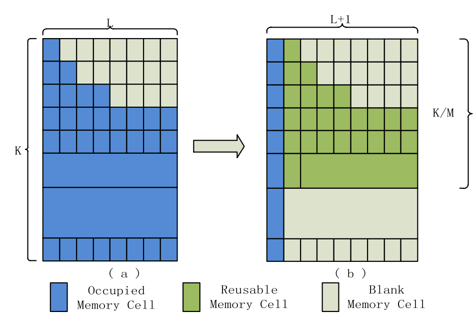

Fig. 2. Sketch of the occupied memory cells under different schemes.

decoder should carry out CRC detection for its sub-block, select and store a certain sequence and use this sequence in the next decoding procedure. The selected sequence should satisfy the following two conditions: 1), the corresponding CRC is satisfied; 2), the likelihood value is the largest among all sequences which fulfill the first condition. Otherwise the decoding fails.

C. Analysis on Memory Space and Decoding Delay

For original SCL decoding, all L survival paths with the size of $K \times L$ bits must be saved in the memory cells until the Kth information bit is obtained. Figure 2(a) shows memory cells for keeping L survival paths with the size of near $K \times L$ for single-CRC aided SCL decoder.

After the multi-CRC aided SCL decoder has completed the CRC check on each sub-block and stored the selected sequences, these memory cells may be cleared and new values can be written into these memory cells in the next decoding process. Here we make a rough estimation of memory space reduction. If the information sequence is divided into M sub-blocks, meaning that the length of each sub-block is

$\frac{K}{M}$, then $\frac{K}{M} \times L$ memory cells are required to keep L survival paths in the decoding process of each sub-block. The decoder needs extra K memory cells to store selected estimation bits in the whole decoding, and the total memory space needed is $\frac{K}{M} \times L + K$ as shown in Fig. 2(b). Dealing with the same information block length, the required memory space is $K \times L$ for single CRC aided SCL decoder. The ratio of memory space for multi-CRC over single CRC method is $(\frac{1}{M} + \frac{1}{L})$. When $M > 2$ and $L > 4$, the value $\frac{1}{M} + \frac{1}{L}$ is less than 0.5 and one can save memory space for more than 50%.

Next, we discuss the average delay on output decoded bits for two different schemes. Multi-CRC aided SCL decoder can output a sequence of a certain sub-block once the decoding of this sub-block is finished. We define average decoding delay T_d as follows.

$$T_d = \frac{1}{M}(t_{d_1} + t_{d_2} + \cdots + t_{d_M}), \tag{1}$$

where t_{d_i} is the decoding delay of the ith sub-block. It is clear that the decoding delay is proportional to the decoding computation complexity, so we just analyze the computation complexity of two different SCL decoding schemes. For original single CRC aided SCL decoding with code length N and list size L, computation complexity is $O(L \times N \log N)$ and decoding delay can be represented as

$$T_{d_s} = c \cdot L \times N \log N, \tag{2}$$

where c is a constant factor. On the other hand, if we separate information block into M segments with equal length and neglect the cost of CRC check, then the output delay of ith sub-block is

$$t_{d_i} = c \cdot L \times \frac{i \times N}{M} \log\left(\frac{i \times N}{M}\right). \tag{3}$$

The average decoding delay of multi-CRC aided SCL decoding can be expressed as

$$T_{d_m} = \frac{1}{M}\sum_{i=1}^{M} c \cdot L \frac{i \times N}{M} \log\left(\frac{i \times N}{M}\right). \tag{4}$$

The ratio $\frac{T_{d_m}}{T_{d_s}}$ is less than one and the value is about 0.5 when $M = 4$ and $N = 1024$, meaning that decoding delay can be reduced about 50%.

D. Discussion on Performance of BLER

In this section, the performance in terms of BLER is evaluated via simulations. We consider BPSK modulation over the AWGN channel for two different decoding schemes. We divide an information block into $M = 4$ segments and use CRC-4 $(x^4 + x + 1)$ as error detection code for each sub-block in multi-CRC aided SCL decoding. For comparison, CRC-16 $(x^{16} + x^{15} + x^2 + 1)$ is appended to the information block and decoded by single-CRC aided SCL decoding. Figure 3 presents BLER performance of (1024,512) polar codes for two SCL decoding schemes with list size 4 and 32. CA-SCL denotes single-CRC aided SCL decoding, and multi-CRC SCL denotes multi-CRC aided SCL decoding. From Fig. 3, we can see that there is a degradation in BLER performance.

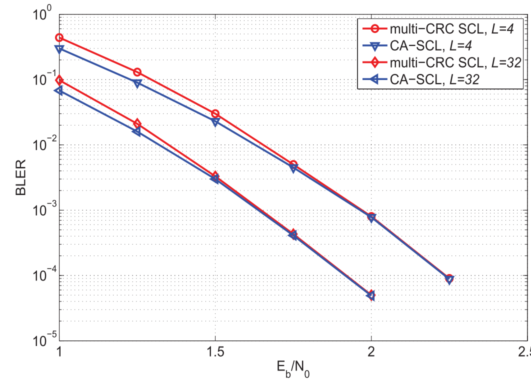

Fig. 3. BLER of single-CRC and multi-CRC aided SCL decoding.

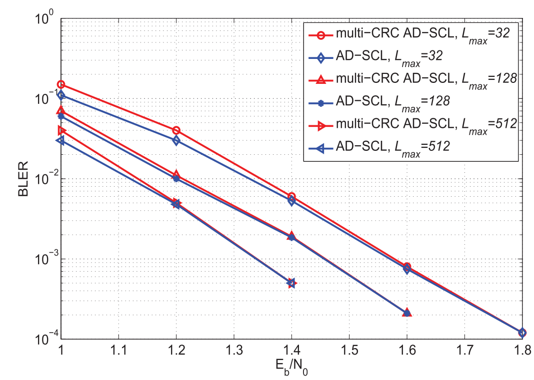

Fig. 4. BLER of single-CRC and multi-CRC aided AD-SCL decoding.

However, when the SNR reaches 1.5 dB or higher, the degradation becomes negligible. BLER performance comparison of (2048,1024) polar code of two schemes under adaptive list decoding is shown in Fig. 4 with maximum list size $L_{max} \in \{32, 128, 512\}$. AD-SCL means single-CRC aided AD-SCL decoding, and multi-CRC AD-SCL stands for multi-CRC aided AD-SCL decoding. We observe again a degradation of BLER performance but this degradation is negligible when SNR is more than 1.4 dB.

For multi-CRC aided SCL decoding scheme, it is noted that its performance is sub-optimal and may not approach the ML performance with increasing list size L due to making decision at earlier level. Also, the detection error capability of CRC may be exponentially decreasing with the length of CRC redundancy. The interested reader may refer to [8] for a systematic analysis on the effect of CRC decoding. However, the BLER gap between our proposed multi-CRC scheme and the single-CRC one is very close and upper bounded by a small value. To explain, we first note the two conditions for the selection of the survival path: the reliability from the decoder as well as CRC value. When SNR increases, the error probability from the decoder decreases accordingly and falls into the range of CRC detection with short redundant length, leading to a small BLER gap between two schemes.

IV. HARQ Based on Multi-CRC Polar Codes

In this section, we apply multi-CRC polar codes to hybrid automatic repeat request (HARQ) to aid retransmission. HARQ is an efficient technique for reliable communication over channels with unknown or time-varying channel state, and is combination of forward-error correction (FEC) with an ARQ protocol. Our proposed HARQ scheme based on multi-CRC polar codes will be presented and the corresponding throughput will be evaluated in the following.

A. HARQ System Model Based on Multi-CRC Polar Codes

The HARQ scheme based on multi-CRC polar code is similar to that in [9] in which every retransmitted sub-block consists of information bits only. In our proposed system and that in [8], a CRC block is appended to each information block, or sub-block, and the resultant K-length packet is encoded by (N, K) polar code. In the receiver, positive/negative acknowledgement (ACK/NACK) is fed back to transmitter based on CRC detection result over an error-free feedback link. If an NACK followed the corresponding information is received, an information sub-sequence of U bits from original information sequence is generated based on the channel reliability computed using density evolution [10]. The U bits of most unreliable information bits form the redundancy packets and will be sent in the next round. The reliability of channel should be updated each time after transmission of redundancy packets. Repeat the operation until an ACK is fed back or the maximum retransmission round N_t is satisfied.

In HARQ system using single CRC aided decoding, the U most unreliable information bits are selected based on the whole information sequence with size K if CRC detection fails. For HARQ using multi-CRC aided polar code, the U bits from sub-block with the size $\frac{K}{M}$ in which CRC detection fails are selected. Thus, those bits that are not correctly decoded are retransmitted with higher probability using multi-CRC to help retransmission. For performance comparison, we simulate the throughput of HARQ using single CRC aided and multi-CRC aided decoding.

B. Simulation and Discussion

Figure 5 shows simulation results for multi-CRC polar code and single CRC polar code under the SC decoding, list decoding with list size 4 and 32, respectively. The multi-CRC polar codes are with information length $K = 512$ and block length $N = 1024$, and four CRC-4 blocks are uniformly-spaced distributed in the information sequence. Similarly, single CRC polar code (1024, 512) with CRC-16 serves as a comparison. We set the maximum retransmission round N_t to be 4 and each selective copy has 128 bits chosen according to density evolution. From Fig. 5, we can see that the proposed scheme leads to a higher throughput.

In the following, we compare these two schemes with regard to their individual throughput. The throughput η is defined as the average number of successfully decoded message bits per transmitted code symbol. ε_i denotes the event "decoding failure occurs at the i-th round", and $p(i) = \Pr(\varepsilon_1, \ldots, \varepsilon_i)$ denotes the joint probability of an event that i consecutive decoding failures occur at 1st,2nd,$\ldots$, i-th round. The throughput can be expressed as

$$\eta = \frac{K\left[1 - p(N_t)\right]}{N + \sum_{i=1}^{N_t-1} U \cdot p(i)}. \tag{5}$$

From (5), it is noted that η increases with a larger $p(i)$, $i = 1, \ldots, N_t$. Since the incorrectly decoded bits are retransmitted with higher probability in multi-CRC scheme, the probability of decoding failure is decreased and the throughput grows at SNR from -4 dB to 0 dB. When SNR increases, the probability of decoding success almost becomes the same as that of single CRC scheme. Further analysis may be carried out based on reference [11] which provides a systematic multiple-packet strategy employing multiple CRCs for HARQ.

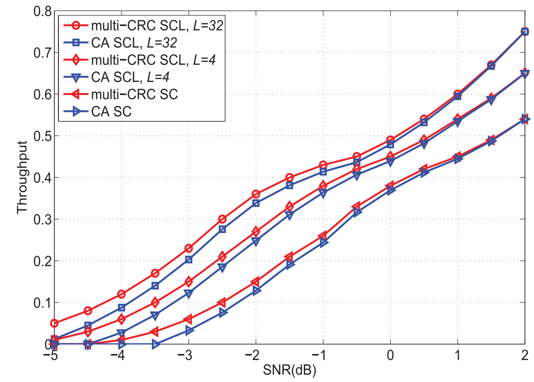

Fig. 5. Throughput of HARQ under different decoding schemes.

V. Conclusion

In this letter, we have proposed a multi-CRC polar code and the corresponding SCL decoding scheme which lead to about 50% reduction of the memory space and decoding delay but with negligible performance loss. Furthermore, we have applied multi-CRC polar code to HARQ scheme and have shown by simulations that the throughput of multi-CRC polar codes is higher than that of single-CRC polar codes.

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